Switch angles in the first quarter

As we have seen so far, trigonometric functions of angles in I quarter are calculated

in the same way as trigonometric functions of sharp angles of a right-angled triangle.

Trigonometric functions arbitrary angle can be expressed through trigonometric functions appropriate angle

in I quarter.

This process is called: **switch angles in the first quarter.**

1) From II to I quarter

Apply a formula for $0 < \alpha < \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha \quad \rightarrow \quad \boxed{\sin\left(90^0+\alpha\right)=\cos\alpha}$$

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha \quad \rightarrow \quad \boxed{\cos\left(90^0+\alpha\right)=-\sin\alpha}$$

$$tg\left(\frac{\pi}{2}+\alpha\right)=-ctg\alpha \quad \rightarrow \quad \boxed{tg\left(90^0+\alpha\right)=-ctg\alpha}$$

$$ctg\left(\frac{\pi}{2}+\alpha\right)=-tg\alpha \quad \rightarrow \quad \boxed{ctg\left(90^0+\alpha\right)=-tg\alpha}$$

and

$$\sin(\pi-\alpha)=\sin\alpha$$
$$\cos(\pi-\alpha)=-\cos\alpha$$
$$tg(\pi-\alpha)=-tg\alpha$$
$$ctg(\pi-\alpha)=-ctg\alpha$$

$or:$

$$\boxed{\begin{array}{l}\sin(180^0-\alpha)=\sin\alpha\\ \cos(180^0-\alpha)=-\cos\alpha\\ tg(180^0-\alpha)=-tg\alpha\\ ctg(180^0-\alpha)=-ctg\alpha\end{array}}$$

Examples:

a) $\sin 115^o = \sin(90^o + 25^o) = \cos 25^o$ also, we can do this:

$\sin 115^o = \sin(180^o + 65^o) = \sin 65^o$

Of course, we have already seen"connections"in the first quarter, and we know that $\cos 25^o = \sin 65^o$. You can use any formula of this two.

b) $\cos\frac{3\pi}{4}=\cos\left(\pi-\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

c) $tg141^o = tg\left(180^o - 39^o\right) = -tg39^o$

d) $ctg101^o = ctg\left(90^o + 11^o\right) = -tg11^o$

2) From III to I quarter

Again we have two groups of the formula:

$$\sin(\pi+\alpha)=-\sin\alpha$$
$$\cos(\pi+\alpha)=-\cos\alpha$$
$$tg(\pi+\alpha)=tg\alpha$$
$$ctg(\pi+\alpha)=ctg\alpha$$

or:

$$\boxed{\begin{aligned}&\sin(180^0+\alpha)=-\sin\alpha\\&\cos(180^0+\alpha)=-\cos\alpha\\&tg(180^0+\alpha)=tg\alpha\\&ctg(180^0+\alpha)=ctg\alpha\end{aligned}}$$

and

$$\sin\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha \rightarrow \boxed{\sin\left(270^0-\alpha\right)=-\cos\alpha}$$
$$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha \rightarrow \boxed{\cos\left(270^0-\alpha\right)=-\sin\alpha}$$
$$tg\left(\frac{3\pi}{2}-\alpha\right)=ctg\alpha \rightarrow \boxed{tg\left(270^0-\alpha\right)=ctg\alpha}$$
$$ctg\left(\frac{3\pi}{2}-\alpha\right)=tg\alpha \rightarrow \boxed{ctg\left(270^0-\alpha\right)=tg\alpha}$$

Examples:

a) $\sin\frac{4\pi}{3}=\sin\left(\pi+\frac{\pi}{3}\right)=-\sin\frac{\pi}{3}=-\frac{\sqrt{3}}{2}$

b) $\cos 207^o=\cos\left(180^o+27^o\right)=-\cos 27^o$

c) $tg263^o=tg\left(270^o-7^o\right)=ctg7^o$

d) $ctg\frac{7\pi}{6}=ctg\left(\pi+\frac{\pi}{6}\right)=ctg\frac{\pi}{6}=\sqrt{3}$

3) From IV to I quarter

$$\sin\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$$
$$\cos\left(\frac{3\pi}{2}+\alpha\right)=-\sin\alpha$$
$$tg\left(\frac{3\pi}{2}+\alpha\right)=-ctg\alpha$$
$$ctg\left(\frac{3\pi}{2}+\alpha\right)=-tg\alpha$$

and

$$\sin(2\pi-\alpha)=-\sin\alpha$$
$$\cos(2\pi-\alpha)=\cos\alpha$$
$$tg(2\pi-\alpha)=-tg\alpha$$
$$ctg(2\pi-\alpha)=-ctg\alpha$$

If you look at a negative angle $(-\alpha)$:

sin(-α)= - sinα **cos(-α)=cosα** **tg(-α)= tgα** **ctg(-α)= -ctg** α

Examples:

a) $\sin 307^o = \sin(270^o + 37^o) = -\cos 37^o$

b) $\cos(-30^o) = \cos 30^o = \frac{\sqrt{3}}{2}$

c) $tg\frac{11\pi}{6} = tg\left(-\frac{\pi}{6}\right) = -tg\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

d) $ctg\left(-\frac{\pi}{3}\right) = -ctg\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

We have already noted that:

$$\sin(\alpha + 2k\pi) = \sin\alpha$$
$$\cos(\alpha + 2k\pi) = \cos\alpha$$

for any whole number k .

Examples:

a) $\sin 1170^o =$?

$1170^o - 360^o = 810^o$

$810 - 360^o = 450^o$

$450^o - 360 = 90^o$

So: $\sin 1170^o = \sin 90^o = 1$ or we can write: $\sin 1170^o = \sin(90^o + 3 \cdot 2\pi) = \sin 90^o = 1$

b) $\cos 780^o = ?$

$780^o - 360^o = 420^o$

$420^o - 360^o = 60^o$

So: $\cos 780^o = \cos 60^o = \frac{1}{2}$ $\longrightarrow$ $\cos 780^o = \cos(60^o + 360^o) = \cos 60^o = \frac{1}{2}$

For tangent and cotangent is:

$$tg(\alpha + k\pi) = tg\alpha \quad \rightarrow \quad tg(\alpha + k \cdot 180^0) = tg\alpha$$
$$ctg(\alpha + k\pi) = ctg\alpha \quad \rightarrow \quad ctg(\alpha + k \cdot 180^0) = ctg\alpha$$

So the basic period of functions tgx and $ctgx$ is $T = \pi$ ($T = 180^o$)

Examples:

a) $tg750^o = ?$

$$750^o - 180^o = 570^o$$
$$570^o - 180^o = 390$$
$$390^o - 180^o = 210^o$$
$$210^o - 180^o = 30^o$$
$$tg750^o = tg30^o = \frac{\sqrt{3}}{3}$$

b) $ctg(-1110^o) = -ctg1110^o = -ctg30^o = -\sqrt{3}$ because: $1110^o = 6 \cdot 180^o + 30^o$

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1. Simplify: $\dfrac{\sin 750^o \cdot \cos 390^o \cdot tg1140^o}{ctg405^o \cdot \sin 1860^o \cdot \cos 780^o}$

Solution:

First, we use the formula and all “move” in I quarter!

$$\sin 750^o = \sin(30^o + 2 \cdot 360^o) = \sin 30^o = \frac{1}{2}$$
$$\cos 390^o = \cos(30^o + 360^o) = \cos 30^o = \frac{\sqrt{3}}{3}$$
$$tg1140^o = tg(60^o + 6 \cdot 180^o) = tg60^o = \sqrt{3}$$

$$ctg405^o = ctg(45^o + 2 \cdot 180^o) = ctg45^o = 1$$
$$\sin 1860^o = \sin(60^o + 5 \cdot 360^o) = \sin 60^o = \frac{\sqrt{3}}{3}$$
$$\cos 780^o = \cos(60^o + 2 \cdot 360^o) = \cos 60^o = \frac{1}{2}$$

So:

$$\frac{\sin 750^o \cdot \cos 390^o \cdot tg1140^o}{ctg405^o \cdot \sin 1860^o \cdot \cos 780^o} = \frac{\cancel{\frac{1}{2}} \cdot \cancel{\frac{\sqrt{3}}{2}} \cdot \sqrt{3}}{1 \cdot \cancel{\frac{\sqrt{3}}{2}} \cdot \cancel{\frac{1}{2}}} = \sqrt{3}$$

2. Simplify: $\dfrac{\cos\frac{17\pi}{6}\cdot\sin\frac{7\pi}{3}\cdot tg\frac{17\pi}{4}}{ctg\frac{10\pi}{3}\cdot\cos\frac{7\pi}{4}\cdot\sin\frac{8\pi}{3}}$

Solution:

Similarly, as in the previous task, all transferred to I quarter.

$$\cos\frac{17\pi}{6}=\cos\frac{17\cdot180^o}{6}=\cos510^o=\cos150^o=\cos(180^o-30^o)=-\cos30^o=-\frac{\sqrt{3}}{2}$$

$$\sin\frac{7\pi}{3}=\sin\left(\frac{\pi}{3}+2\pi\right)=\sin\frac{\pi}{3}=\sin60^o=\frac{\sqrt{3}}{2}$$

$$tg\frac{17\pi}{4}=tg\left(\frac{\pi}{4}+\frac{9\pi}{3}\right)=tg\left(\frac{\pi}{4}+4\pi\right)=tg\frac{\pi}{4}=tg45^o=1$$

$$ctg\frac{10\pi}{3}=ctg\left(\frac{\pi}{3}+\frac{9\pi}{3}\right)=ctg\left(\frac{\pi}{3}+3\pi\right)=ctg\frac{\pi}{3}=ctg60^o=\frac{\sqrt{3}}{3}$$

$$\cos\frac{7\pi}{4}=\cos\frac{7\cdot180^o}{4}=\cos315^o=\cos(-45^o)=\cos45^o=\frac{\sqrt{2}}{2}$$

$$\sin\frac{8\pi}{3}=\sin\left(\frac{2\pi}{3}+\frac{6\pi}{3}\right)=\sin\left(\frac{2\pi}{3}+2\pi\right)=\sin\frac{2\pi}{3}=\sin\frac{2\cdot180^o}{3}=\sin120^o=\sin(90^o+30^o)=$$

$$=\cos30^o=\frac{\sqrt{3}}{2}$$

Change these values in the task:

$$\frac{\cos\frac{17\pi}{6}\cdot\sin\frac{7\pi}{3}\cdot tg\frac{17\pi}{4}}{ctg\frac{10\pi}{3}\cdot\cos\frac{7\pi}{4}\cdot\sin\frac{8\pi}{3}}=\frac{-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}\cdot1}{\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}=-\frac{3}{\sqrt{2}}=-\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{3\sqrt{2}}{2}$$

3) Prove identity: $\dfrac{\sin\alpha-2\sin(\pi-\alpha)}{\cos(\pi-\alpha)-\cos\alpha}=\dfrac{1}{2}tg\alpha$

Proof:

In identity move from one side and transformed it , until you reach the other side.

Because is:

$$\sin(\pi-\alpha)=\sin\alpha$$
$$\cos(\pi-\alpha)=-\cos\alpha$$

$$\frac{\sin\alpha-2\sin(\pi-\alpha)}{\cos(\pi-\alpha)-\cos\alpha}=\frac{\sin\alpha-2\sin\alpha}{-\cos\alpha-\cos\alpha}=\frac{-\sin\alpha}{2\cos\alpha}=$$
$$=\frac{1}{2}\frac{\sin\alpha}{\cos\alpha}=\frac{1}{2}tg\alpha$$

4) Prove identity: $$\frac{\cos\left(\frac{3\pi}{2}-\alpha\right)ctg\left(\frac{\pi}{2}+\alpha\right)\cos(-\alpha)}{\cos(2\pi+\alpha)tg(\pi-\alpha)}=-\sin\alpha$$

Proof:

We will use:

$$\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$$
$$ctg\left(\frac{\pi}{2}+\alpha\right)=-tg\alpha$$
$$\cos(-\alpha)=\cos\alpha$$
$$\cos(2\pi+\alpha)=\cos\alpha$$
$$tg(\pi-\alpha)=-tg\alpha$$

So: $$\frac{\cos\left(\frac{3\pi}{2}-\alpha\right)ctg\left(\frac{\pi}{2}+\alpha\right)\cos(-\alpha)}{\cos(2\pi+\alpha)tg(\pi-\alpha)}=\frac{(-\sin\alpha)\,\cancel{(-tg\alpha)}\,\cancel{(\cos\alpha)}}{\cancel{(\cos\alpha)}\,\cancel{(-tg\alpha)}}=-\sin\alpha$$